New aspects of the purity and information of an entangled qubit pair.

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Abstract. In this article, we investigate the dynamics of the purity of the entangled of 2 two-level atoms interacting with a single quantized electromagnetic field. We show that how the purity of the qubit pairs depends on initial state of the atomic system. It is found that the superposition case is the best choice to generate entangled states with high purity and hence high entanglement. It is clear that the purity of one qubit can purified by the expense of the other pair through the phenomena of purity swapping. The mean photon number plays an important role in increasing the purity. The robustness of the quantum channel is investigate in the presences of individual attacks, where we study the separability of these channels and evaluate its fidelity. Finally, we use these channels to perform the original coding protocol by using theses partial entangled states. We find that Bob can gets the codded information with reasonable percentage. The inequality of security is tested, where we determine the interval of times in which Alice and Bob can communicate secure. These intervals depend on the type of error and the structure of initial atomic system.

keywords: Qubit, Entanglement, Purity, Dense coding.

1. Introduction

Evolution of atoms interacting with a coherent field is an interesting topic in quantum optics [1]. The Jaynes- Cummings model, JCM, is the simplest model describes the interaction of a single two -level atom with a single quantized electromagnetic field and can be realized experimentally [2]. Nowadays, related studies are concerned with quantum information [3, 4] and computations [5]. In this work, we consider simple model of a two two-level atoms interacting with a single mode radiation field. Due to the interaction, the behavior of the two atoms swap from separable to entangled and vis versa. In our calculations, we consider only the entangled case of the two qubit state. On the other hand, since most of quantum information processing requires entangled state with high degree of purity and hence a degree of entanglement, we focus here on the effect of the structure of the initial atomic system on the degree of purity i.e, we answer the questions, what is the best choice of the initial atomic state to generate an entangled state with high purity? and how the purities of the entangled qubit pairs and their individual subsystems evolves with time?. Since one can use these states as channels to code information, we investigate the robustness of theses channel in the presences of the individual attacks. Finally, we use theses quantum channel to perform original coding protocol [6]. Also, the possibility of sending a secure information between two users is investigated, where we determine the secure interval of times, in which the users can communicate secure.

The article is arranged as follows: In sec.2, we describe the model and its time evolution. Also we introduce the final state of the density operator in the computational basis. The behavior of the purity is the subject of subsection 3, where the calculations are performed only when the density operator behaves as an entangled state. Sec.4 is devoted to investigate the robustness of the quantum channel is in the presences of eavesdropper, where we consider Eve applies the individual attacks strategy. In sec. 5.1, we calculate the average amount of information gained from the coded information. Also the possibility of sending a secure information is investigate in sec.5.2. Finally we discuss our result in sec.6.

2. The system and its evolution

The Hamiltonian which describes a system of a two two -level atoms each consisting of states $|e\rangle$ and $|g\rangle$ coupled to a single mode radiation field in the rotating wave approximation is given by

$$H = \omega(a^\dagger a + \sum_{i=1}^{2} \sigma_z^i) + \sum_{i=1}^{2} [\lambda_i (a^\dagger \sigma_-^{(i)} + \sigma_+^{(i)} a)] \tag{1}$$

where $a(a^\dagger)$is the annihilation (creation) operator of the field mode, $\sigma_\pm^i$ and σ_z^i the parameters λ_i are the atom-field coupling constant ω, is the atomic transitions and the field mode frequency. The first term in Eq.(1) represents the free-Field and the

non-interacting atoms, while the second term stands for the interaction Hamiltonian, H_{int}. This model has been solved analytically for some special cases and for a general case [7]. In this work, we introduce a direct solution for this model. For simplicity, we consider the case of identical atoms i.e.$\lambda_1 = \lambda_2$. Assume that the cavity field is initially prepared in a coherent state $|\psi(0)\rangle_f = \sum_{n=0}^{\infty} q_n |n\rangle$, where $q_n = exp(-\frac{\bar{n}}{2})\frac{\bar{n}^{\frac{n}{2}}}{\sqrt{n!}}$. For the atomic system, we consider the first atom is in its excited state i.e. $|\psi(0)\rangle_1 = |e\rangle_1$ and the other atom is in a superposition state $|\psi(0)\rangle_2 = a|e\rangle_2 + b|g\rangle_2$, where 1 stands for the first atom, 2 for the second atom and $|a|^2 + |b|^2 = 1$. So, we can write the initial state of the two atoms as $|\psi(0)\rangle_{12} = |e\rangle_1 \otimes (a|e\rangle_2 + b|g\rangle_2)$ and consequently the initial state of the combined system is given by,

$$|\psi_0\rangle = \sum_{n=0}^{\infty} q_n \Big\{ |n\rangle \otimes (a|e\rangle_1 \otimes |e\rangle_2 + b|e\rangle_1 \otimes |g\rangle_2) \Big\}. \tag{2}$$

At any time $t > 0$, the atom-field state is described by the state,

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle. \tag{3}$$

Since we are interested in the behavior of the purity and information contained in the entangled two atoms, we trace over the field state. After some straightforward calculations one can get the density operator of the two atoms explicitly in the computational basis as $|00\rangle$, $|10\rangle$, $|01\rangle$ and $|11\rangle$:

$$\begin{aligned}\rho(t)_{12} = {} & |c_n^{(1)}|^2|00\rangle\langle 00| + c_n^{*(1)}c_{n+2}^{(2)}|10\rangle\langle 00| + c_n^{*(1)}c_{n+1}^{(3)}|01\rangle\langle 00| + c_n^{*(1)}c_{n+2}^{(4)}|11\rangle\langle 00| \\ & + c_n^{(1)}c_{n+2}^{*(2)}|00\rangle\langle 10| + |c_n^{(2)}|^2|10\rangle\langle 10| + c_n^{*(2)}c_n^{(3)}|01\rangle\langle 10| + c_n^{*(2)}c_{n+1}^{(4)}|11\rangle\langle 10| \\ & + c_n^{(1)}c_{n+1}^{*(3)}|00\rangle\langle 01| + c_n^{(2)}c_n^{*(3)}|10\rangle\langle 01| + |c_n^{(3)}|^2|01\rangle\langle 01| + c_n^{*(3)}c_{n+1}^{(4)}|11\rangle\langle 01| \\ & + c_n^{(1)}c_{n+2}^{*(4)}|00\rangle\langle 11| + c_n^{*(2)}c_{n+1}^{(4)}|10\rangle\langle 11| + c_n^{(3)}c_{n+1}^{*(4)}|01\rangle\langle 11| + |c_n^{(4)}|^2|11\rangle\langle 11|,\end{aligned} \tag{4}$$

where,

$$\begin{aligned} c_n^{(1)}(t) = {} & -\sum_{n=0}^{\infty} q_n^2 \frac{\gamma_n}{\sqrt{\mu_n}} \left(\frac{b\beta_n}{\sqrt{\mu_n}}(1-\cos t\sqrt{\mu_n}) + ia\sin t\sqrt{\mu_n} \right), \\ c_n^{(2)}(t) = {} & \sum_{n=0}^{\infty} q_n^2 \left(\frac{2a}{\mu_n}(\beta_n^2+\gamma_n^2)\sin^2 t\sqrt{\mu_n} + a\cos t\sqrt{\mu_n} - i\frac{2\beta_n}{\sqrt{\mu_n}}\sin t\sqrt{\mu_n} \right), \\ c_n^{(3)}(t) = {} & -\sum_{n=0}^{\infty} q_n^2 \frac{\sin t\sqrt{\mu_n}}{\sqrt{\mu_n}} \left(\frac{2a}{\sqrt{\mu_n}}(\beta_n^2+\gamma_n^2) + b\beta_n \right), \\ c_n^{(4)}(t) = {} & \sum_{n=0}^{\infty} q_n^2 \frac{\cos t\sqrt{\mu_n}}{\sqrt{\mu_n}} \left(\frac{2b\gamma_n^2}{\sqrt{\mu_n}}(\mu_n - 2\gamma_n^2) - ia\beta_n \right), \end{aligned} \tag{5}$$

with

$$\gamma_n = \sqrt{n+1}, \quad \beta_n = \sqrt{n+2}, \quad \mu_n = 2(\gamma_n^2 + \beta_n^2), \quad \tau = \lambda t. \tag{6}$$

3. Purity

Most of quantum information tasks require entangled pure state to be performed. This aim is a difficult to achieve in reality due to the dechorence. In this section, we try to answer the following question. How close the generated entangled state of the two atoms to purity?. On the other hand, since the interaction depends on the initial state of the two atoms, we study how the states of the atoms behave individually. In our study, we consider the impurity of the two atoms ρ_{12}, ρ_1 for the first atom and ρ_2 for the second atom. Let us define the degree of impurity as

$$\eta_i = 1 - tr\rho_i^2, \tag{7}$$

where $i = 1, 2, 12$ see for example [8]. For the density operator ρ_{12} the degree of impurity is given by,

$$\begin{aligned}\eta_{12} = 1 - \Big\{&|c_n^{(1)}|^2\Big[|c_n^{(1)}|^2 + 2(|c_{n+1}^{(2)}|^2 + |c_{n+1}^{(3)}|^2 + |c_{n+2}^{(4)})|^2)\Big] \\ &+ |c_n^{(2)}|^2\Big[|c_n^{(2)}|^2 + 2(|c_n^{(3)}|^2 + |c_{n+1}^{(4)}|^2)\Big] \\ &+ |c_n^{(3)}|^2\Big[|c_n^{(3)}|^2 + 2(|c_{n+1}^{(4)}|^2)\Big] + |c_n^{(4)}|^4\Big\},\end{aligned} \tag{8}$$

for the first atom is,

$$\begin{aligned}\eta_1 = 1 - \Big\{&\left(|c_n^{(1)}|^2 + |c_n^{(3)}|^2\right)^2 \\ &+ 2\Big[|c_{n+1}^{(2)}|^2|c_n^{(1)}|^2 + |c_{n+1}^{(4)}|^2|c_n^{(3)}|^2|c_{n+1}^{(2)}|^2|c_n^{*(1)}|^2|c_n^{(3)}|^2|c_{n+1}^{(4)}|^2 \\ &+ |c_{n+1}^{(4)}|^2|c_n^{*(3)}|^2|c_n^{(1)}|^2|c_{n+1}^{*(2)}|^2 + \left(|c_n^{(2)}|^2 + |c_n^{(4)}|^2\right)^2\Big]\Big\},\end{aligned} \tag{9}$$

and for the second atom is,

$$\eta_2 = 1 - \Big\{\left(|c_n^{(1)}|^2 + |c_n^{(2)}|^2\right)^2$$

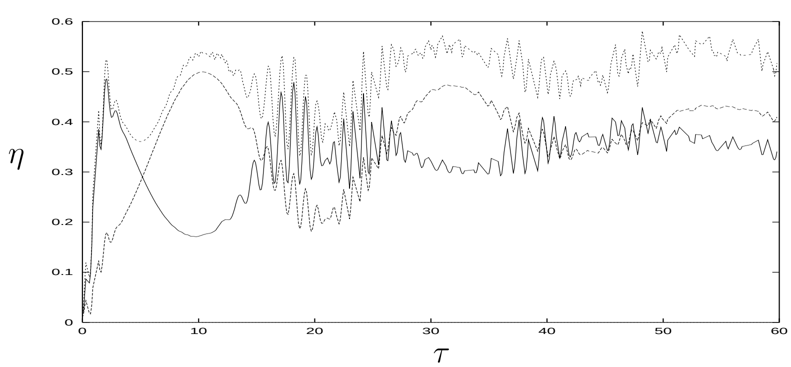

Figure 1. The degree of impurity as a function of the scaled time τ with $= \bar{n} = 10$. The dot line for ρ_{12} the solid line for ρ_1 and the dash line for ρ_2. The atomic system is initially prepared in a superposition product state $a|ee\rangle + b|eg\rangle$, where $a = 0.5$ and b=$\sqrt{1-a^2}$.

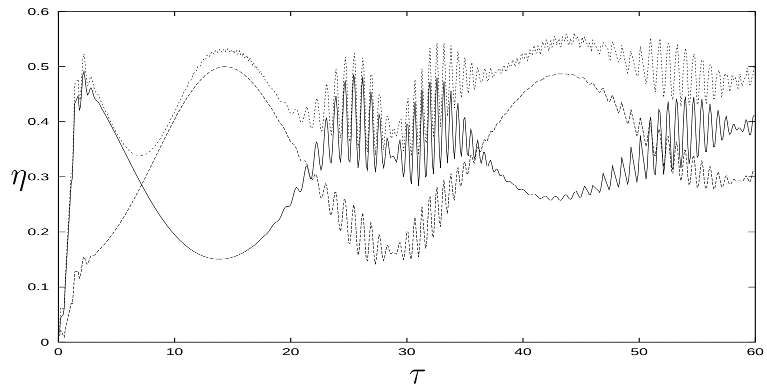

Figure 2. The same as Fig.1, but $\bar{n} = 20$.

$$+ 2\Big[|c_n^{(1)}|^2|c_{n+1}^{(3)}|^2 + |c_n^{(2)}|^2|c_{n+1}^{(4)}|^2|c_{n+1}^{(3)}|^2|c_n^{*(1)}|^2|c_n^{(2)}|^2|c_{n+1}^{*(4)}|^2$$
$$+ |c_{n+1}^{(4)}|^2|c_n^{*(2)}|^2|c_n^{(1)}|^2|c_{n+1}^{*(3)}|^2 + \big(|c_n^{(3)}|^2 + |c_n^{(4)}|^2\big)^2\Big]\Big\}, \qquad (10)$$

where $c's$ are given by (5). Fig.1, shows the behavior of the degree of impurity for three states, the first atom ρ_1, the second atom ρ_2 and the state of the two atoms ρ_{12}. In this investigation, we consider only the *entangled* states of ρ_{12} and its corresponding subsystems ρ_1 and ρ_2. We consider that the initial state of the system is in a superposition state and the mean photon number $\bar{n} = 10$. From this figure, it is clear that when the impurity of the first atom is maximum, the impurity of the second one has a minimum impurity. This phenomena is called impurity *swapping*. On other words, one can say that the purity of one qubit is increased at the expense of the other qubit. The behavior of η for ρ_{12} and ρ_2 is the same, but the degree of purity is different. For some intervals of time we obtain an entangled stated of purity, $(1-\eta) > 0.90$. This is an important result for quantum information and computation, where these applications need pure states with a high purity to be performed. Also we can see that as the time evolves the degree of purity decreases. This explain why the degree of entanglement decreases as the time evolves.

Fig.2, shows the effect of the mean photon number on the degree of purity. It is clear that as one increases $\bar{n}$. the degree of purity increases. This is clear by comparing Fig.1 and Fig.2, in addition to the usual behavior of Rabi oscillations is seen. The only difference in these figures is that the number of oscillations is different. This is due to the lost of phase to the separable state, where in our calculation, we consider only the *entangled states*. As $\bar{n}$ increases , a large modulation in the oscillations and a clear shift from the usual Rabi oscillations occurs. Also the degree of purity increases as the values of $\bar{n}$ increases and the amplitudes of Rabi oscillations decrease [15].

In Fig.3, we consider that the atomic system is initially prepared in excite state. In this case the degree of purity decreases and hence the degree of entanglement. By comparing Fig.2 and Fig.3, we can conclude that one can generate an entangled state with high degree of purity and consequently high degree of entanglement, if the atomic

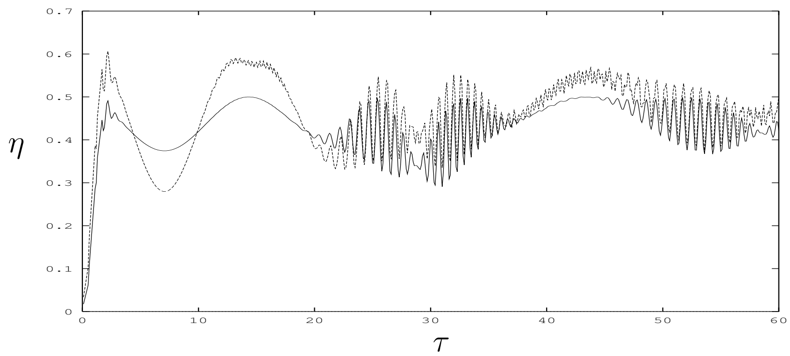

Figure 3. The same as Fig.2, but the atomic system is prepared initially in exited state.

system is prepared initially in a superposition product states. It is clear that the degree of purity depends on the structure of the initial states of the two atoms. From this, we can argue that, starting from a superposition state as an initial state for the atomic system is recommended as a best one of the nest choices.

4. Robustness of the quantum channel

In this section, we try to investigate the stability of the quantum channel in the presence of the eavesdropper. Let us restrict ourself on the individual attacks. For this strategy, Eve, has the ability to access Alice's qubit and make a projective measurements along a certain basis to get the encoded information. Also, we assume that Eve sends another qubit to Bob by applying a unitary operators $I, \sigma_x, \sigma_y, \sigma_z$ randomly on the original state. As an example if Eve applies σ_z on the travelling qubit of Alice, then Bob will get a new state is defined by.

$$\begin{aligned}\tilde{\rho}(t)_{12} = {} & |c_n^{(1)}|^2|00\rangle\langle 00| - c_n^{*(1)}c_{n+2}^{(2)}|01\rangle\langle 00| + c_n^{*(1)}c_{n+2}^{(3)}|01\rangle\langle 00| - c_n^{*(1)}c_{n+2}^{(4)}|11\rangle\langle 00| \\ & - c_n^{(1)}c_{n+2}^{*(2)}|00\rangle\langle 10| + |c_n^{(2)}|^2|10\rangle\langle 10| + c_n^{*(2)}c_n^{(3)}|01\rangle\langle 10| + c_n^{*(2)}c_{n+1}^{(4)}|11\rangle\langle 10| \\ & + c_n^{(1)}c_{n+1}^{*(3)}|00\rangle\langle 01| + c_n^{(2)}c_n^{*(3)}|10\rangle\langle 01| + |c_n^{(3)}|^2|01\rangle\langle 01| - c_n^{*(3)}c_{n+1}^{(4)}|11\rangle\langle 01| \\ & - c_n^{(1)}c_{n+2}^{*(4)}|00\rangle\langle 11| + c_n^{*(2)}c_{n+1}^{(4)}|10\rangle\langle 11| - c_n^{(3)}c_{n+1}^{*(4)}|01\rangle\langle 11| + |c_n^{(4)}|^2|11\rangle\langle 11|,\end{aligned} \tag{11}$$

Now, let us study the behavior of the channel $\tilde{\rho}_{12}$ from the separability point of view. For this aim we use the positive partial transpose criteria (PPT) [9]. This criteria states that, a density operator is separable if its partial transpose is non- negative. If this criteria is violated then the density operator is entangled. In Fig.4, we plot this criteria for two different values of the initial atomic system. In Fig.(4a), we plot the PPT criterion when Eve operates by σ_x on Alice's qubit. Starting from an atomic system prepared initially in a product excited state, one gets an entangled state once the interaction time goes on. But due to the instability, this state turns into a separable state in a small interval of the interaction time, $\tau = [1.4 - 1.6]$. Then the state behaves as astable entangled state. On the other hand, if the atomic system is initially prepared

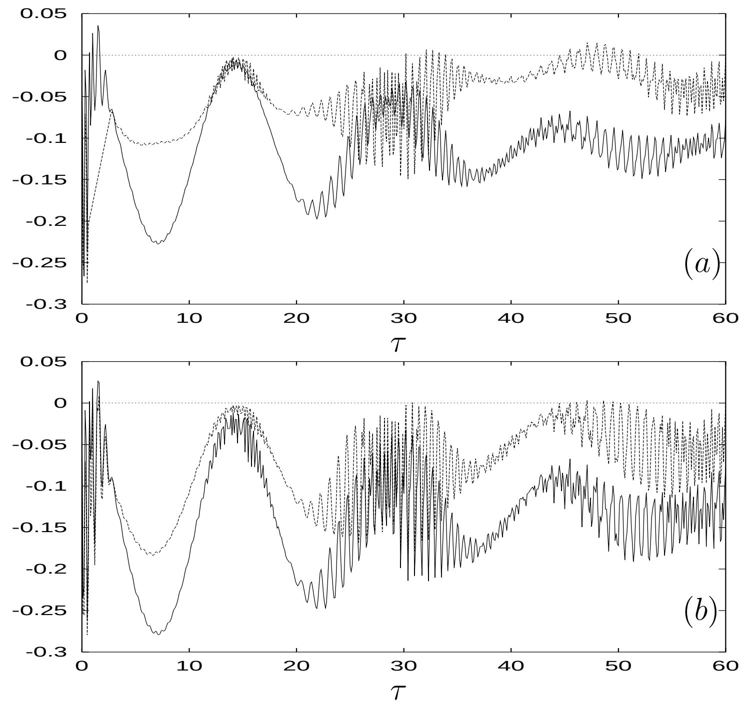

Figure 4. The PPT criterion of the quantum channel, ρ_{12} with $\bar{n} = 20$ when Eve sends another qubit to Bob. the solid curves and the dot curves for the atomic system initially prepared in excited and superposition product states respectively. (a) If Eve applies σ_x on Alice's qubit. (b)If Eve applies σ_z on Alice's qubit.

in a superposition product state, the generated entangled state is instable in along range of the interaction time, $\tau = [45.4-48.9]$. In this interval it turns into a separable several times. In Fig.($4b$), we study the separability of a new channel, if Eve sends another qubit to Bob by rotating by Alice's qubit by σ_z. In this case the generated state is a stable entangled state expect in a small rang of the interaction time $\tau = [1.4 - 1.6]$ where the atomic system is prepared initially in a product excited state. But starting by atomic system in a superposition product states one can generate a more robust entangled state and never turns into a separable state.

One may ask a question here: how much the new channel related to the original one?. To answer this question we discuss the *fidelity* of the quantum channel. In this context the fidelity is defined as [10]

$$F = tr\{\rho_{12} U_i \otimes I_2 \rho_{12}^i U_i^\dagger \otimes I_2\} \tag{12}$$

where $i = 0, 1, 2, 3$, $U_0 = I_1$, is the unitary operator for Alice qubit, I_2 for Bob qubit, $U_1 = \sigma_x, U_2 = i\sigma_y$ and $U_3 = \sigma_z$.

In Fig.($5a$), we plot the fidelity of the new quantum channel $\tilde{\rho}_{12}$, where we consider that Eve applies σ_x on Alice's qubit. In this case Bob, will get a state with a small fidelity. The fidelity decreases for atomic system prepared in a superposition product states and reaches zero which coincides with Fig.($5a$), where the state is separable. For this strategy one can expect that, Eve can distill more information from Alice's massage.

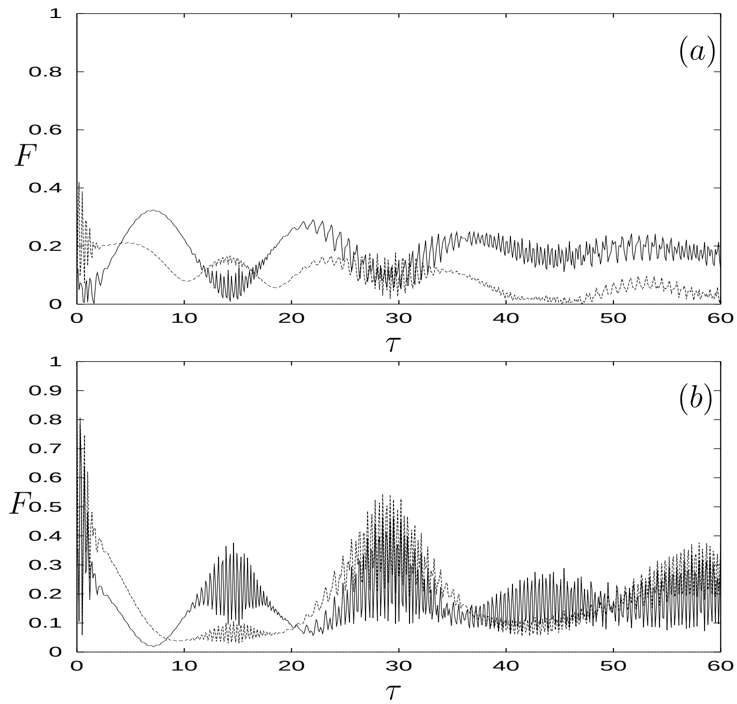

Figure 5. The fidelity,F, of the new channel when Eve send another qubit to Bob. The solid and the dot curves are for the atomic system is prepared initially in an excited product state and superposition product states respectively and $\bar{n} = 20$. (a) If Alice qubit is rotated by σ_x. (b) If Alice qubit is rotated by σ_z.

In Fig.($5b$), the fidelity is plotted for the other strategy of Eve, i.e when she operates by σ_z on Alice's qubit. In this case the fidelity is better than the previous case which depicted in Fig.($5a$) and never reach to zero. This means that in this case the generated states are robust against this strategy.

5. Quantum communation

In this section, we try to employ the generated entangled state to send a secure information between the partner Alice and Bob. Assume that Alice has the first atom while the second at Bob's hand. Let us use a resonator to entangle the two atoms. For this purpose the atoms are brought into the cavity for a certain lapse of time. Then they are removed from the cavity and shared between (distant) partners (Alice and Bob). The entangled atom pairs are distributed and can be used for quantum communication. We use the dense coding protocol to send the codded information between Alice and Bob. Also let us assume that the eavesdropper, Eve, would have access the atoms after they have distributed to the partners, and she will use the individual attacks strategy. In the following subsection we investigate the dense coding protocol. Also we give a secure analysis of this communication in Sec.5.2.

5.1. Dense coding Protocol

In this subsection, we use the generated entangled state to perform the original dense coding protocol. Since it has been proposed by Bennett and Wiesner[6], there are several versions of this protocol [11]. Achieving dense coding protocol via cavity has been investigated in Ref[12]. Also the dense coding protocol by using partial entangled state is investigated by Mozes et al [13]. The most recent practical implementation of dense coding has been performed by Xing and Gong[14]. The original coding protocol can be described in two steps:

(i) Assume that Alice and Bob share a pair of entangled particles ρ_{12}. Alice can encode two classical bits in her qubit by using one of the local unitary operations, U_i(defined after Eq.(12)) given by . If, we assume that she performs these operations randomly, then with a probability p_i, she codes her information in the state,

$$\rho_{cod} = \sum_{i=0}^{3}\Big\{p_i U_i \otimes I_2 \rho_{12}^i U_i^\dagger \otimes I_2\Big\}. \tag{13}$$

(ii) Alice sends her qubit to Bob, who tries to decodes the information. To perform this task he makes a joint measurement on the two qubits, where the two qubits are at his disposal. The maximum amount of information which Bob can extract from Alice's massage is bounded by

$$I_{Bob} = \mathcal{S}\Big(\sum_{i=0}^{3} p_i \rho_{12}^{(i)}\Big) - \sum_{i=0}^{3} p_i \mathcal{S}(\rho_{12}^{(i)}), \tag{14}$$

where $\mathcal{S}(.)$, is the Von Numann entropy. In Fig.6, we plot the average amount of information gained by Bob, where we consider that Alice has used the unitary operator with equal probability, i.e $p_i = \frac{1}{4}$. From this figure we may conclude that in some intervals of time Bob can get more information from the coded massage. On the other hand, if the Alice and Bob start from atomic system in a superposition state and use the generated state to code information, it will be better than if they start from an excited state of the atomic system.

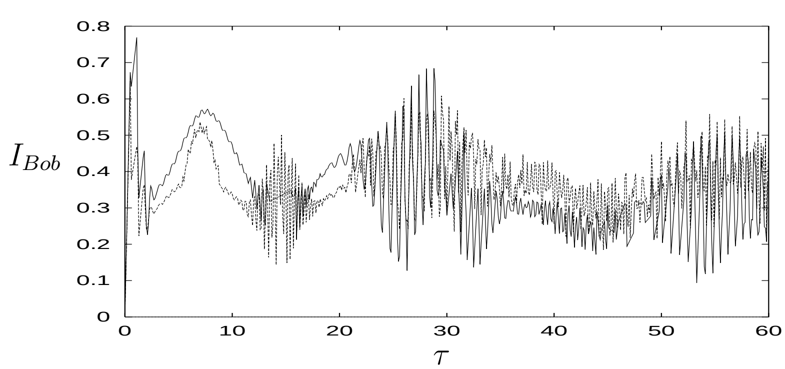

Figure 6. The amount of information decoded by Bob. The dot and the solid curves for the atomic system is initially prepared in a superposition and excited states respectively where $\bar{n} = 20$.

5.2. Security analysis

As we have mentation before that Eve will use the individual attacks strategy. In this case the eavesdropper, Eave can accesses Alice's atom and resends another one to Bob. Due to the presences of the eavesdropper the fidelity, F, of the shared state between Alice and Bob decreases. In this case the Bob's error rate, (Disturbance) is defined by $D = 1 - F$, where $F = \frac{1}{4}\sum_{i=0}^{3} tr\{\rho_{12}U_i \otimes I_2 \rho_{12}^i U_i^\dagger \otimes I_2\}$. As the disturbance increases, the fidelity of the state between Alice and Bob that govern the probability that they will accept the transmitted state decreases. On the other hand Eve's probability of correctly guessing more information is increases. In this case the relevant mutual information between Alice and Eve as a function of Bob's error is given by [16],

$$I_{AE} = log_2(2) + (1 - D)log_2(1 - D) + Dlog_2 D. \tag{15}$$

The users Alice and Bob can communicate secure and hence they establish a secret key, if the Bob's error rate satisfies the inequality,

$$(1 - D)log_2(1 - D) + Dlog_2(D) \leq -\frac{1}{2}. \tag{16}$$

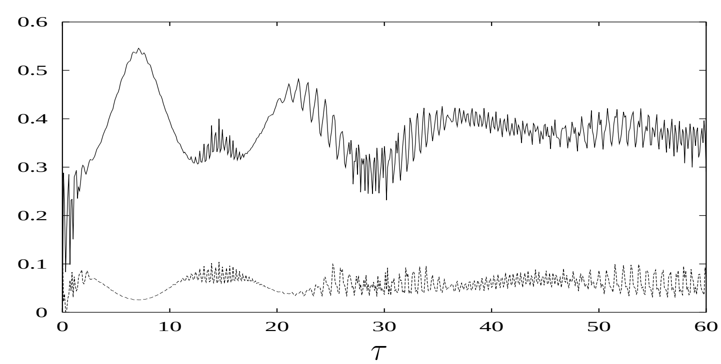

Figure 7. The average amount of the mutual information between Alice and Eve I_{AE}(dot curves) and between Alice and Bob I_{AB}(solid curve). The atomic system is initially prepared in a excited product states where $\bar{n} = 20$.

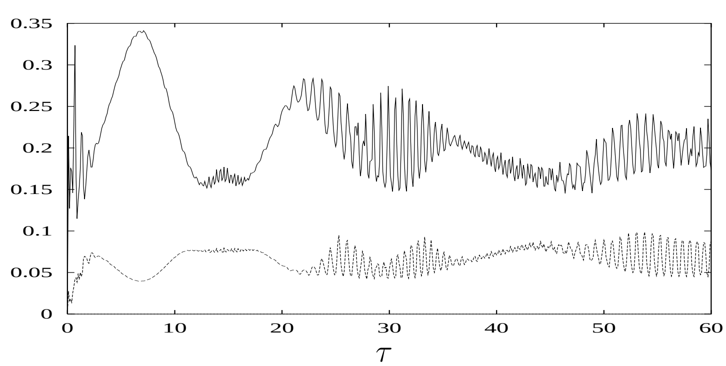

Figure 8. The average amount of the mutual information between Alice and Eve I_{AE}(dot curves) and between Alice and Bob I_{AB}(solid curve). The atomic system is initially prepared in a superposition product states where $\bar{n} = 20$.

This inequality gives bounds on Bob's permissible error rate in the case of individual attacks.

Fig.7 shows the behavior of the average amount of the mutual information between Alice, Eve (I_{AE}) and Alice, Bob (I_{AB}), where we assume that the initial atomic system is prepared in a excited product case. This information is plotted when the inequality of the security, (16) is obeyed. It is clear that in some intervals of time, the mutual information between Alice and Eve, I_{AE} decreases while the mutual information between Alice and Bob, I_{AB} increases. Although in some intervals of time I_{AE} increases in the expanse of I_{AB}, Alice and Bob still communicate in a secure way. Fig.8, shows the behavior of I_{AB} and I_{AE}, where the atomic system is prepared in a superposition product state and the secure inequality (16) is satisfied. By comparing Fig.(7) and Fig.(8), we can see that I_{AB}, which is depicted in Fig.(7) is much larger than that in Fig.(8). So, starting from atomic system prepared in excited product state, the partner can communicate safely and the secure information is much larger.

In Fig.(9), a special case is considered where we assume that Eve, causes a phase error on the travelling qubit. In this figure, the behavior of the mutual information I_{AB}, I_{AE} and the inequality of security (16) are plotted. It is clear that, for some intervals of time $I_{AE} < I_{AB}$ and the inequality of security is obeyed. So in these intervals Alice and Bob can communicate in a secure way. Although $I_{AE} < I_{AB}$ in some intervals of time, as an examples $[5.2 - 8.2]$ and $[20.2 - 23.]$, but the channel is insecure. In this case Alice and Bob will not accept the probability of the transmitted information. For some other intervals of time $I_{AE} > I_{AB}$ and the inequality of security is violated. This means that through these intervals the channel is insecure and the partner, Alice and Bob can not communicate safely.

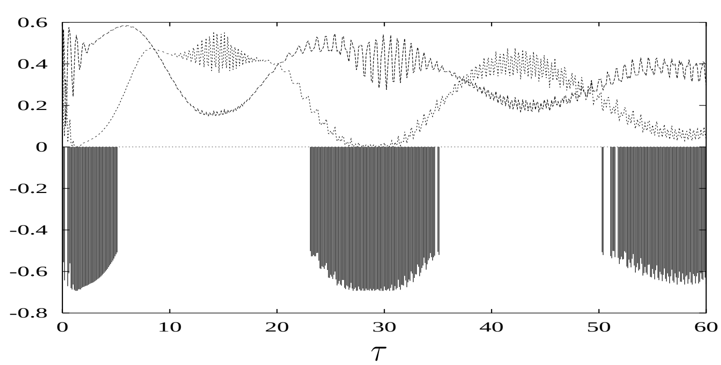

Figure 9. The mutual information between Alice and Eve I_{AE}(dot curve) and between Alice and Bob, I_{AB}(dash curve). The solid curves represent the secure inequality when it is obeyed. Eve applies σ_z on the travelling qubit , $\bar{n} = 20$ and the atomic system prepared initially in a superposition state.

6. conclusions

In this paper, we have considered the system of two two-level atoms interacting with a cavity field. It is shown that, generating entangled states with high degree of purity and hence high degree of entanglement depends on the initial state of the atomic system. For our system this is achieved with the superposition state. The dynamics of the purity of the individual atoms show the swapping phenomena. The purity of one of qubit can be purified at the expense of the other qubit through the dynamics of the purity swapping. Also, as one increases the values of the mean photon number, the degree of purity increases and consequently the degree of entanglement.

The robustness and the fragile of the channel are investigated in the presences of the individual attacks. We find that for some strategy of Eve the channel is fragile. In this case Eve can distill some information from the coded massage. On other strategy, the channel is more robust and the eavesdropper can not get more information. Also if we start with atomic system prepared initially in excited state, one can generate entangled a more robust entangled states.

Finally, we employ the generated entangled state to perform the original dense coding protocol. It is possible to send a codded massage from Alice to Bob with reasonable fidelity. This fidelity depends on the structure of the initial atomic system. It has been shown that choosing the atomic system initially prepared in a superposition state is much better.

We show that the average amount of the coded information can be transmitted between the users securely, where the inequality of security is obeyed. It is clear that, although the average codded information is better if the partner start with a superposition state, the possibility of the secure communication is decreases. Also, an example is given, where we assume that Eve applies the shift error operator. We determine the intervals of time in which the channel is secure and the partner can use it safely, where the inequality of security is tested.

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